

# Modelling and Solving the Capacitated Location-Routing Problem with Simultaneous Pickup and Delivery Demands

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## Abstract

In this work, the capacitated location-routing problem with simultaneous pickup and delivery (CLRP-SPD) is considered. This problem is a more realistic case of the capacitated location-routing problem (CLRP) and belongs to the reverse logistics of the supply chain. The problem has many real-life applications of which some have been addressed in the literature such as management of liquid petroleum gas tanks, laundry service of hotels and drink distribution. The CLRP-SPD is composed of two well-known problems; facility location problem and vehicle routing problem. In CLRP-SPD, a set of customers with given delivery and pickup demands should be supplied by a fleet of vehicles that start and end their tours at a single depot. Moreover, the depots and vehicles have a predefined capacity and the objective function is minimizing the route distances, fixed costs of establishing the depot(s) and employing the vehicles. The node-based MIP formulation of the CLRP-SPD is proposed based on the literature of the problem. To solve the model, a greedy clustering method (GCM) is developed which includes four phases; clustering the customers, establishing the proper depot(s), assigning the clusters to depot(s) and constructing the vehicle tours by ant colony system (ACS). The numerical experiments on two sets of test problems with different sizes on the number of customers and candidate depots show the efficiency of the heuristic method with the proposed method in the literature. Finally, performance of the heuristic method to the similar methods in the literature is evaluated by several standard test problems of the CLRP.

**Keywords:** Capacitated location-routing problem; simultaneous pickup and delivery; greedy clustering method; ant colony system.

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## 1. Introduction and Literature Review

In each supply chain, making the good strategies and decisions to reduce the logistic costs is the one of the important issues that should be considered more. In recent years, the efficient, reliable, and flexible decisions on location of depots and the vehicle routes are of vital importance for managers [Zare Mehrjerdi and Nadizadeh, 2013; Tavakkoli-Moghaddam et al.2016]. Logistic cost is usually related to the locating the distributing centers (DC) or depots and routing between the customers and depots by a fleet of vehicles [Zare Mehrjerdi and Nadizadeh, 2016]. Many researchers indicated that if the routes are ignored while locating the depots, the costs of distribution systems might be immoderate [Webb, 1968; Salhi and Rand, 1989; Prins et al.2006]. The location-routing problem (LRP) overcomes this drawback by simultaneously considering the location and routing decisions [Nadizadeh and Kafash, 2017].

The LRP is defined as a special case of vehicle routing problem (VRP) in which there is a need to solve the facility location problem (FLP), simultaneously [Zarandi et al.2011]. Since both problems belong to the class of NP-hard problem, the LRP is also NP-hard problem [Barreto et al.2007; Belenguer et al.2011]. LRP is applicable for a wide variety of fields such as food and drink distribution, newspapers delivery, waste collection, bill delivery, military applications, used oil management, organization of natural disaster, battery swap stations, parcel delivery and various consumer goods distribution [Manzour-al-Ajdad et al.2012; Rath and Gutjahr, 2014; Zhao and Verter, 2014; Yang and Sun, 2015].

In LRP, the customers should only be supplied by a single vehicle; in the other word the vehicle meets every customer once. Each vehicle also starts and ends its tour at a single depot. In the LRP, the proper depot(s) between candidate depots as well as the vehicle tours should be established. The objective is to minimize the total distance of routes as well as fixed depot and vehicle costs [Nadizadeh et

al.2011; Escobar, 2014]. Furthermore, the capacitated location-routing problem (CLRP) is a version of LRP that constrained by the vehicles and depots capacities [Nadizadeh and Nasab, 2014].

Laporte is the first researcher who discusses and classifies the LRP models [Laporte, 1988]. Min et al. [Min et al.1998] also review the LRP literature using a hierarchical classification based on the problem characteristics such as the number of depots, the capacity of depots and vehicles, the form of the objective function and etc. More recently, Nagy and Salhi [Nagy and Salhi, 2007] perform a comprehensive literature review on the LRP models, solution approaches, application areas and some future works. Since the solution times increase exponentially with an increase in the size of the problem, most papers in field of LRP and CLRP have focused on only new solution approaches that are often based on heuristic or meta-heuristic approaches [Nadizadeh et al.2017]. Some reviews on solution approaches of CLRP exist in literature that can be found in [Duhamel et al.2010; Derbel et al.2012; Zarandi et al.2013].

Recently, two review researches are carried out to survey recent publications of LRP's Models; Prodhon and Prins analyzed the literature on the standard LRP and the extensions such as several distribution echelons, multiple objectives or uncertain data [Prodhon and Prins, 2014]. They also compared the results of state-of-the-art meta-heuristics on standard sets of instances for the classical LRP, the two-echelon LRP and the truck and trailer problem. Drexl and Schneider presented paper discussed variants and extensions of the standard LRP, which include problems with stochastic and fuzzy data, multi-period planning horizons, continuous location in the plane, multiple objectives, more complex demands or route structures, such as pickup and delivery demands or routes with load transfers, and inventory decisions [Drexl and Schneider, 2015].

In this paper, the capacitated location-routing problem with simultaneous pickup and delivery (CLRP-SPD) is observed. The CLRP-SPD is a variant of the CLRP where the vehicles are not

**Table 1. Related works of the LRP-SPD**

| <b>Author(s)</b>                            | <b>Year</b> | <b>Contributions and/or approaches.</b>  |
|---|-------------|--|
| [Karaoglan et al.2011]                      | 2011        | Branch-and-cut algorithm   |
| [Karaoglan et al. 2012]                     | 2012        | Simulated annealing  |
| [Yu and Lin, 2014]                          | 2014        | Multi-start simulated annealing  |
| [Huang, 2015]                               | 2015        | LRP with pickup–delivery routes and stochastic demands + Tabu search   |
| [Rahmani et al.2015]                        | 2015        | Two-Echelon Multi-products LRP-SPD + Local Search approach   |
| [Yu and Lin, 2016]                          | 2015        | Simulated annealing  |
| [Rahmani et al.2016]                        | 2016        | Two-Echelon Multi-products LRP-SPD + Nearest neighbour and insertion approaches  |
| [Ghatreh Samani and Hosseini-Motlagh, 2017] | 2017        | Two-Echelon LRP-SPD with fuzzy demands + Hybrid of SA and GA   |
| [Wang and Li, 2017]                         | 2017        | LRP-SPD with heterogeneous fleet and time windows + Hybrid heuristic algorithm (genetic algorithm + variable neighborhood algorithm) |

only required to deliver goods to customers but also to pick up some goods from the customers, simultaneously. The CLRP-SPD arises in context of reverse logistics and there are various real cases, such as distribution of bottled drinks, chemicals, LPG (liquid petroleum gas) tanks, laundry service of hotels and etc. where the customers are typically visited for a double service. In the case of the bottled drinks for instance, full bottles are delivered to customers and empty ones are brought back either for reuse or for recycling [Nadizadeh, 2017]. In the CLRP-SPD, the problem is more complicated than CLRP because of the fluctuating loads on the vehicle along a route. In the CLRP, the total load of each route must not exceed the capacity of the vehicle. But in CLRP-SPD, the net change (decrease or increase) on the vehicle load at each customer must be monitored by the vehicle capacity [Catay, 2010]. As a result, the CLRP-SPD can reduce to a CLRP after some changes [Karaoglan et al.2012]. Because the CLRP is NP-hard, the CLRP-SPD is also NP-hard.

LRP-SPD, a branch of LRP, was firstly introduced by Karaoglan et al. [Karaoglan et al.2011]. Although the LRP has been studied extensively in the literature, the LRP-SPD has received very little attention from researchers so far. Table 1 summarizes the related works on LRP-SPD, describing their main contributions and/or approaches. Karaoglan et al. [Karaoglan et al.2011] presented a mathematical

formulation for the CLRP-SPD and proposed an effective branch-and-cut algorithm for solving it. Their algorithm composed of several valid inequalities and a local search based on simulated annealing (SA) to obtain upper bounds. Finally to evaluate the proposed algorithm, they solved a large number of benchmark instances, derived from the literature, in a reasonable computation time. In next work, Karaoglan et al. [Karaoglan et al.2012] suggested two polynomial-size mixed integer linear programming formulations for the CLRP-SPD and a number of valid inequalities to strengthen the formulations. While their first formulation was a node-based formulation, the second one was a flow-based formulation. Furthermore, they proposed a two-phase heuristic approach based on SA, to solve the CLRP-SPD. They also generated the initial solutions by two initialization heuristics. Consequently, computational results showed that the flow-based formulation performs better than the node-based formulation in terms of the solution quality and the computation time on small-size problems.

Yu and Lin [Yu and Lin, 2014] proposed a multi-start simulated annealing (MSA) algorithm for solving LRP-SPD which incorporates multi-start hill climbing strategy into simulated annealing framework. The MSA algorithm is tested on some benchmark instances to verify its performance. Their results indicated that the multi-start strategy can

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significantly enhance the performance of traditional single-start simulated annealing algorithm. Wang and Li [Wang and Li, 2017] studied on a low carbon for LRP with heterogeneous fleet, simultaneous pickup-delivery and time windows. They designed a two-phased hybrid heuristic algorithm to solve the problem. Firstly, the concept of temporal-spatial distance with genetic algorithm is used to cluster the customer points to construct the initial path. Then, variable neighborhood search algorithm is applied for local search. Computational results showed that the initial solution considering temporal-spatial distance has obvious advantages in the efficiency of the algorithm and the quality of the solution.

So far, many heuristic approaches have expanded in the literature of the LRP, which can be categorized in four main groups namely, sequential, clustering, iterative, and hierarchical methods. In sequential methods, in first step, the summation of depot to customer distances is minimized and then, the VRP is solved based upon the location of depots. The clustering-based methods, first create clusters for the customers, then, either solve the VRP for each candidate depot, or solve the traveling salesman problem (TSP) to find the best location of depots. In iterative heuristics, VRP and FLP sub-problems are solved iteratively, feeding information from one phase to the other. In hierarchical method, location of depots is the main problem and routing is a subordinate problem [Nagy and Salhi, 2007]. This paper presents a new efficient solution approach that belongs to clustering-based methods, based on the above classification. In fact, a greedy clustering method (GCM) is proposed to solve the CLRSP-SPD in four phases. Since a greedy search algorithm is used for clustering the customers in first phase, the proposed method is called “greedy clustering method”. In second phase, among a set of candidate depots, the most appropriate one(s) are selected to be established. The third phase allocates the clusters to depot(s), and finally, ant colony system (ACS) is applied to set up the best routes between the depot(s) and the assigned clusters in fourth phase.

The remainder of this paper is organized as follows; In Section 2, problem definition with an example of CLRSP-SPD including the mathematical formulation is given. Details of the proposed method are presented in section 3. In section 4, the computational results of numerical experiments are reported. Finally, conclusion and future directions of the paper are presented in section 5.

## 2. Problem Definition and Formulation

There are three entities in the CLRSP-SPD that linked together. First, a number of candidate depots which have limited capacities. Second, some of the customers who have specific demands where consist of two parts: the receiving including shipping goods from / to the depot by a vehicle. Third, unrestricted number of fleet of homogeneous vehicles which have a predefined capacity should serve the customers. In the CLRSP-SPD, each vehicle is used only in one route and starts and finishes its route at the same depot. Moreover, the total vehicle load at any point of the route should not exceed the vehicle capacity. On the other hand, each customer is served by exactly one vehicle and the total pickup and delivery load of the customers assigned to a depot should not exceed the capacity of the depot. The problem is to determine the locations of depot(s), the assignment of customers to the opened depot(s) and tour of vehicles with a minimum total cost [Karaoglan et al.2012].

As mentioned previously, in the CLRSP, the feasibility of a route can be determined by checking whether the sum of its customer demands does not exceed the vehicle’s capacities. In contrast, the feasibility of a CLRSP-SPD route depends crucially on the sequence of visitation of the customers. In the example shown in Figure 1, the route  $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0$  is feasible, but the routes  $0 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 0$  or  $0 \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow 0$  are not. This fact suggests the use of extended formulations in CLRSP-SPD against CLRSP, where auxiliary variables are used to enforce route feasibility.

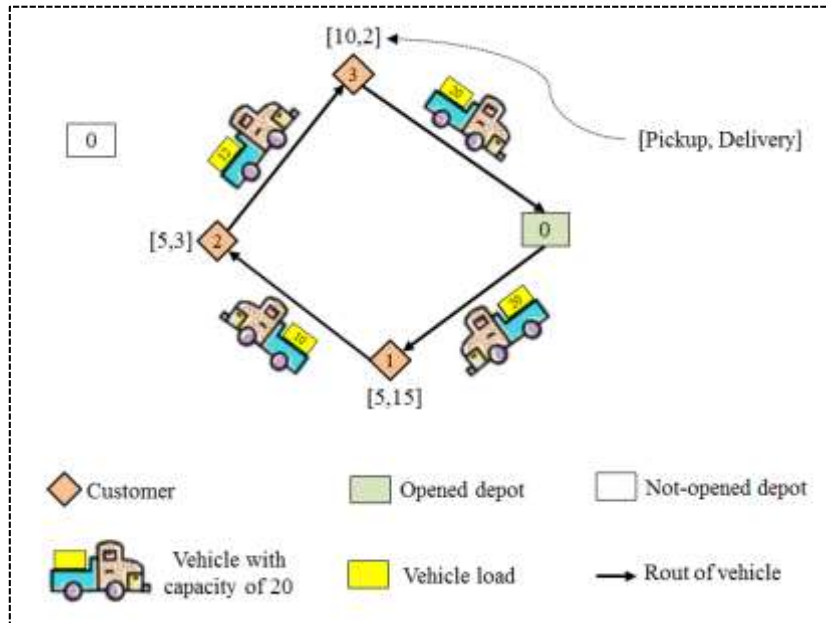


Figure 1. Illustrative example for the CLRSP-SPD.

The node-based Mixed Integer Programming (MIP) formulation for the CLRSP-SPD is proposed as follow. The formulation is adapted to Karaoglan et al. [Karaoglan et al.2012] with small changes. Let  $G = (N,A)$  be a complete directed network where  $N = N_0 \cup N_C$  is a set of nodes in which  $N_0$  and  $N_C$  represent the potential depot nodes and customers, respectively, and  $A = \{(i, j): i, j \in N\}$  is the set of arcs. Each arc  $(i, j)$  has a nonnegative cost (distance)  $c_{ij}$  that is based on Euclidian distance and triangular inequality holds (i.e.,  $c_{ij} + c_{jk} \geq c_{ik}$ ). A capacity  $W_k$  and a fixed cost  $F_k$  are associated with each potential depot  $k \in N_0$ . Moreover, a capacity  $Q$  and fixed operating cost  $f$  are linked with an unlimited fleet of homogeneous vehicles. Each customer  $i \in N_C$  has pickup ( $p_i$ ) and delivery ( $d_i$ ) demands, so that  $0 \leq d_i, p_i \leq Q$ . The variables used in the formulation of CLRSP-SPD are given as follows:

Decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if a vehicle goes directly from} \\ & \text{customer } i \text{ to customer } j (\forall i, j \in N) \\ 0 & \text{otherwise} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if depot } k \text{ is open } (\forall k \in N_0) \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ik} = \begin{cases} 1 & \text{if customer } i \text{ is assigned to} \\ & \text{depot } k (\forall i \in N_C, \forall k \in N_0) \\ 0 & \text{otherwise} \end{cases}$$

Additional variables:

$U_i$ : delivery load on vehicle just before serving customer  $i$  ( $\forall i \in N_C$ )

$V_i$ : pickup load on vehicle just after serving customer  $i$  ( $\forall i \in N_C$ )

The node-based MIP formulation of the CLRSP-SPD is given as follows:

$$\text{Minimize } \sum_{k \in N_0} F_k y_k + \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} + \sum_{k \in N_0} \sum_{i \in N_C} f x_{ki} \quad (1)$$

Subject to:

$$\sum_{j \in N} x_{ij} = 1 \quad \forall i \in N_C \quad (2)$$

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$$\sum_{j \in N} x_{ji} - \sum_{j \in N} x_{ij} = 0 \quad \forall i \in N \quad (3) \quad y_k \in \{0,1\} \quad \forall k \in N_0 \quad (19)$$

$$\sum_{k \in N_0} z_{ik} = 1 \quad \forall i \in N_C \quad (4)$$

$$x_{ik} \leq z_{ik} \quad \forall i \in N_C, \forall k \in N_0 \quad (5)$$

$$x_{ki} \leq z_{ik} \quad \forall i \in N_C, \forall k \in N_0 \quad (6)$$

$$x_{ij} + z_{ik} + \sum_{m \in N_0, m \neq k} z_{jm} \leq 2 \quad \forall i, j \in N_C, i \neq j, \forall k \in N_0 \quad (7)$$

$$\sum_{i \in N_C} d_i z_{ik} \leq W_k y_k \quad \forall k \in N_0 \quad (8)$$

$$\sum_{i \in N_C} p_i z_{ik} \leq W_k y_k \quad \forall k \in N_0 \quad (9)$$

$$U_j - U_i + Qx_{ij} + (Q - d_i - d_j)x_{ji} \leq Q - d_i \quad \forall i, j \in N_C, i \neq j \quad (10)$$

$$V_j - V_i + Qx_{ij} + (Q - p_i - p_j)x_{ji} \leq Q - p_j \quad \forall i, j \in N_C, i \neq j \quad (11)$$

$$U_i + V_i - d_i \leq Q \quad \forall i \in N_C \quad (12)$$

$$U_i \geq d_i + \sum_{j \in N_C, j \neq i} d_j x_{ij} \quad \forall i \in N_C \quad (13)$$

$$V_i \geq p_i + \sum_{j \in N_C, j \neq i} p_j x_{ji} \quad \forall i \in N_C \quad (14)$$

$$U_i \leq Q - (Q - d_i) \left( \sum_{k \in N_0} x_{ik} \right) \quad \forall i \in N_C \quad (15)$$

$$V_i \leq Q - (Q - p_i) \left( \sum_{k \in N_0} x_{ki} \right) \quad \forall i \in N_C \quad (16)$$

$$x_{ij} \in \{0,1\} \quad \forall i, j \in N \quad (17)$$

$$z_{ik} \in \{0,1\} \quad \forall i \in N_C, \forall k \in N_0 \quad (18)$$

In this formulation, the objective function (1) represents the sum of the fixed depots location costs, travel costs and the fixed costs of employing vehicles, respectively. Constraints (2) guarantee that each customer should be served within one route only. Constraints (3) state that the number of entering and leaving arcs to each node are equal. Constraints (4) ensure that each customer must be assigned to only one depot. Constraints (5), (6), and (7) eliminate the unallowable routes, i.e. the routes, which do not start and end at the same depot. Constraints (8) and (9) respectively indicate that total delivery and pickup loads on any depot must not exceed the corresponding depot capacity. Constraints (10) and (11) remove subtours and assure that delivery and pickup demands of each customer are satisfied, respectively. Constraints (12) guarantee that the total net load on any customer does not exceed the vehicle capacity. Constraints (13), (14), (15) and (16) express the relation between decision variables and additional variables. It is notable that constraints (10) and (11) with constraints (15) and (16) give exact values to the additional variables on any feasible route, respectively. Finally, constraints (17), (18), and (19) specify the binary variables used in the formulation.

### 3. Proposed greedy clustering method for the CLRP-SPD

A greedy clustering method, named GCM, is presented in this section to solve the CLRP-SPD. In general, GCM is composed of four phases, which is illustrated in Figure 2. In the first phase, customers are clustered using a greedy search algorithm (Figure 2(a)). The nearest customer to the last added customer to the cluster is selected to be included in the cluster. This is the same as to form a tour in Traveling Salesman Problem (TSP), in which the nearest city to the current city (in a “greedy” search algorithm) is selected as next destination. So, the proposed heuristic method is called “greedy clustering method”. Each cluster can include as much customers as so that the total delivery and pickup demands being

less than the capacity of vehicle. In the second phase, the gravity center of each cluster is calculated which is used to select depot(s) among candidate depots (Figure 2(b)). The clusters are allocated to the opened depot(s) in the third step, considering the distance between the depot and the gravity center of clusters as well as the capacity of the opened depot (Figure 2(c)). Finally, in the fourth phase, ACS forms an admissible tour between each cluster and depot (Figure 2(d)).

The problem is initialized by defining a plane comprising the set of customers, depots, and their coordinate points, namely *CUST* and *DEP*, respectively. The heuristic method is repeated for a predefined number of iterations. When the GCM obtained a better solution, it is replaced to the last best known solution. Moreover, since in the first phase of GCM, the first customer at each cluster is selected, randomly, the constituted clusters are different together in each iteration. Thus, the proposed method can search some feasible solutions among all over the solution space. This can help that GCM avoid confining suboptimal solutions. Details of heuristic method are described in following sections.

### 3.1 Clustering the Customers

The first phase of GCM is clustering of the customers. The customers are clustered according to the “greedy search algorithm”. At first, to found a cluster, a customer is selected randomly from the set of non-clustered customers belongs to *CUST*. The algorithm searches for the nearest customer to the last selected customer of the current cluster. The nearest customer is not included to the cluster if either of the following criteria is met: 1) The number of assigned customers to a cluster reached the maximum number of allowed customer per cluster and 2) The total pickup and delivery demand exceeds the remaining capacity of the vehicle. When the number of customer in each cluster reaches to a given number (*N*), there is no opportunity for any of the customers to enter the current cluster even adding its demand to total demand of cluster is less than the *Cap*. This is to balance the number of customers in all clusters, which influences

choosing the depots in next phase, and the final solution. The maximum number of members for a cluster is determined using a trial and error method.

Once a new customer is selected to be included to a cluster, total pickup and delivery demands of current members adding to its new member is compared with the capacity of the vehicle (*Q*). If total demand is less than the *Q*, the new customer is included in current cluster. Otherwise, last selected customer is withdrawn from the cluster. The greedy search algorithm searches for a new customer close to the last added member of the cluster among the non-clustered customers. This procedure helps to use the maximum capacity of a vehicle. The algorithm finds a new cluster if there is no customer to be assigned to current cluster considering the capacity of vehicle and the maximum number of customers per cluster. When there is no non-clustered customer, the process of clustering stops. Figure 3 illustrates the greedy search algorithm.

### 3.2 Establishing the Depot(s)

This phase of GCM searches among potential sites to establish proper depot(s). First, the gravity center of clusters is calculated according to equation (20), in which  $(X_{(I)}, Y_{(I)})$  is the coordinates of gravity center of cluster *I*,  $(x_i, y_i)$  is the coordinates of customer *i*, and *n<sub>I</sub>* is the number of customers assigned to cluster *I*. The gravity center is used as a deputy of the cluster to select the proper depot(s). Choosing the potential site(s) for establishing depot(s) is same as a single facility location problem (SFLP).

$$(X_{(I)}, Y_{(I)}) = \left( \frac{\sum_{i \in I} x_i}{n_I}, \frac{\sum_{i \in I} y_i}{n_I} \right) \quad (20)$$

Second, the sum of distances between the gravity center of the clusters and each potential site, based on Euclidean distance, is calculated by equation (21). In this equation,  $(x^*, y^*)$  is the coordinates of desired potential site among all potential sites. Moreover, *w<sub>j</sub>* is the total Euclidean distance between potential site *j* and the gravity center of clusters,  $(x_j, y_j)$  is the

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coordinates of potential site  $j$ ,  $(a_i, b_i)$  is the coordinates of gravity center of cluster  $i$ ,  $m$  is the number of clusters, and  $DEP$  is the number of potential sites.

$(x^*, y^*)$ : Minimize

$$w_j = \sum_{i=1}^m \left[ (x_j - a_i)^2 + (y_j - b_i)^2 \right]^{1/2} \quad \forall j = 1, \dots, DEP \quad (21)$$

The potential sites are sorted in an ascending order and ranked from 1 to  $DEP$  according to value of equation (21). Then, the top-ranked potential site is selected to establish. As will be mentioned in next step, if the capacity of the current opened depot is unable to fulfill all clusters, the next potential site of the sorted list is selected to serve the remaining clusters. This procedure (i.e., establishing the depot(s)) is repeated until all clusters are covered.

### 3.3. Allocating Clusters to Depot(s)

In this phase, the clusters are respectively allocated to the ranked depots. Each depot serves clusters as many as possible, if the next cluster demand does not exceed the remaining capacity of the depot. To allocate the clusters, the Euclidian distance of gravity center of each cluster to the top-ranked depot is calculated. Afterwards, the unassigned clusters are ranked in an ascending order based on the distance of their gravity centers to the depot. The top-ranked cluster is allocated to the top-ranked depot. If there is an empty capacity for the top-ranked depot, the second-ranked cluster is allocated to the depot. The allocation process to a depot will be finished when there is not enough capacity to allocate new cluster. In this situation, the allocating procedure is repeated for next-ranked depots until all clusters are allocated.

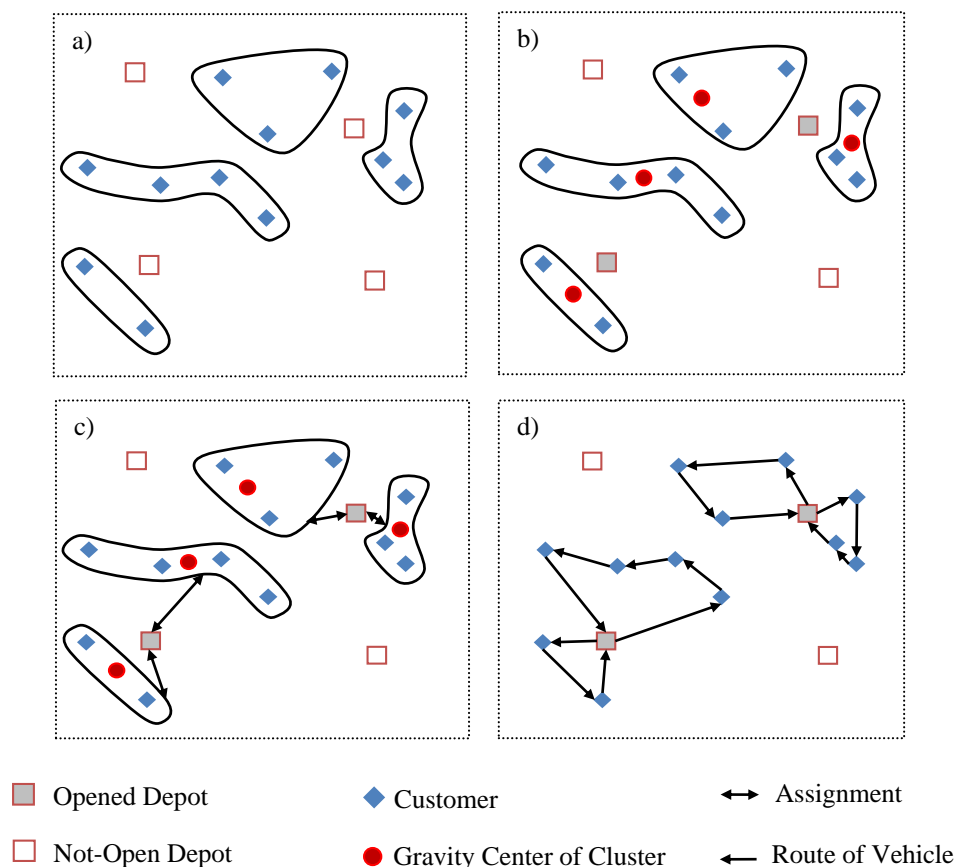


Figure 2. Illustrative example for the greedy clustering method.



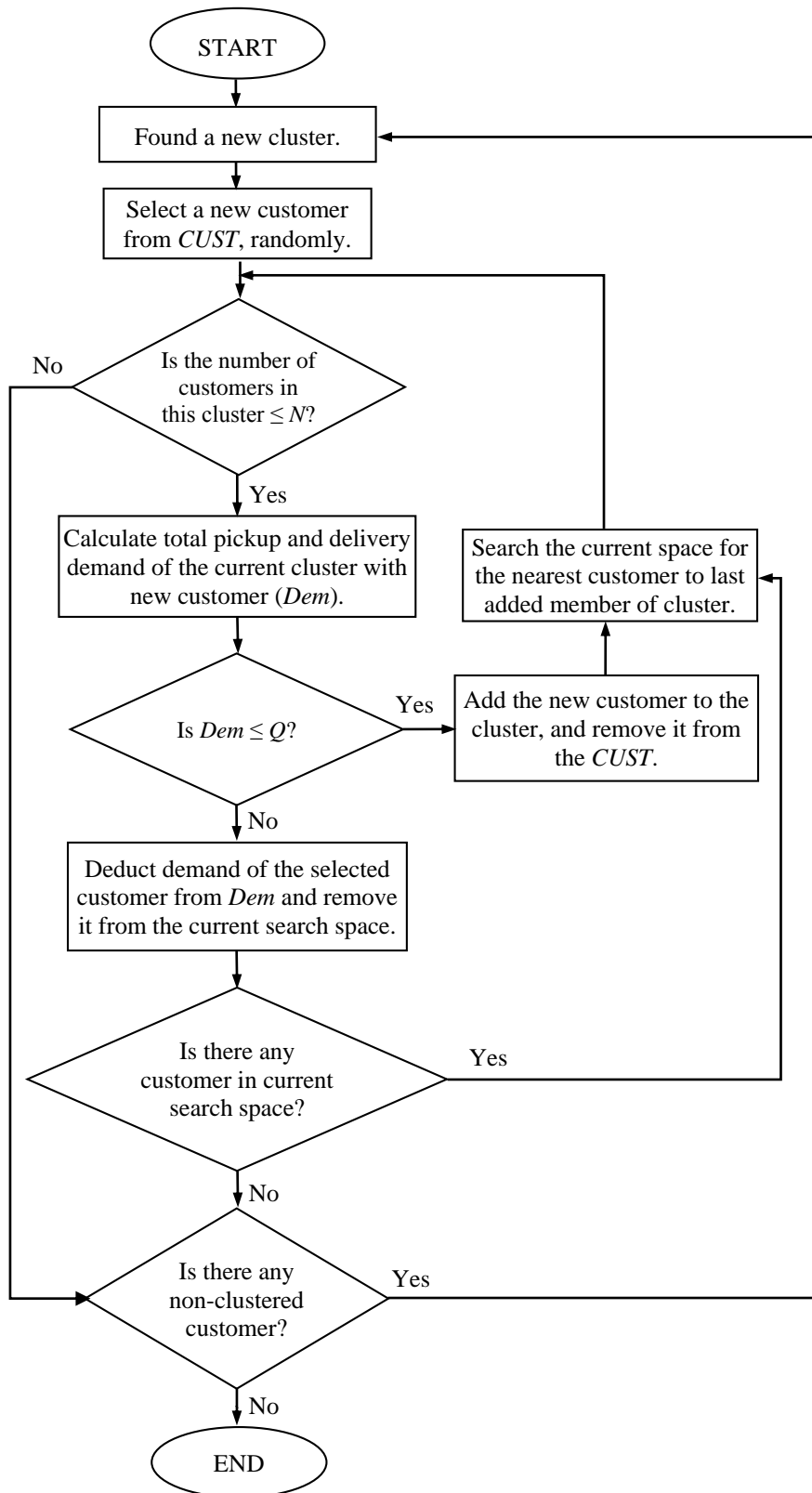


Figure 3. The proposed greedy search algorithm.

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## 3.4 Routing

In the fourth and last phase of GCM, the routing problem for each cluster and its corresponding depot is solved. Each cluster is served by exactly one vehicle, and some vehicles can be supplied by a single depot regarding its capacity. In the routing phase, each cluster with its related depot is considered as a TSP, which is solved by using ant colony system (ACS). ACS is referred to ants' treatment to find food. The ants spread a material called pheromone and put it on their way so that other ants can pass the same route. The pheromone of shorter route increases and therefore, more ants move from that way. Artificial ants construct a solution by selecting a customer to visit sequentially, until all the customers in a route are visited. Ants select the next customer to visit using a combination of heuristic and pheromone information. A local updating rule is applied to modify the pheromone on the selected route, during the construction of a route. When all ants construct their tours, the amount of pheromone of the best selected route and the global best solution, are updated according to the global updating rule. More details on ACS can be found in [Dorigo and Gambardella, 1996; Bouhafs et al.2010; Abolhoseini and Sadeghi-Niaraki, 2017]. Dorigo et al. [Dorigo and Gambardella, 1996] mentioned that the proper parameters' values in their proposed heuristic ACS algorithm are  $\alpha=1$ ,  $\beta=5$  and  $\rho=0.65$ . Hence, these values are used in routing phase of the GCM.

## 4. Computational Results

To evaluate the efficiency of the proposed GCM, a set of computational experiments are carried out. Since no benchmark instances were publicly available for the CLRP-SPD, Karaoglan et al. used two test sets of CLRP generated by Prodhon [Prodhon, 2008] and Barreto [Barreto, 2003]. They applied demand separation approaches based on Salhi and Nagy [Salhi and Nagy, 1999] and Angelelli and Mansini [Angelelli and Mansini, 2002] to change CLRP instances to CLRP-SPD instances. In this paper, a similar approach like in Karaoglan et al. [Karaoglan et al.2011] is

applied to generate test instances which explain briefly as follows.

In Salhi and Nagy's approach, a ratio  $r_i = \min(x_i/y_i; y_i/x_i)$ , where  $x_i$  and  $y_i$  are the coordinates of customer  $i$ , is calculated for each customer  $i$ , and then the delivery and pickup demands are obtained as  $d_i = \lfloor r_i \times q_i \rfloor$  and  $p_i = \lfloor q_i - d_i \rfloor$ , where  $q_i$  is the demand of customer  $i$ . The instances generated by this type are named  $X$ . Similarly, another type of instances, called  $Y$ , is generated by exchanging delivery and pickup demands of each customer. In Angelelli and Mansini's approach, the demand of customer  $i$  is considered as delivery demand ( $d_i = q_i$ ) and the pickup demand is generated by  $p_i = \lfloor (1 - \gamma)q_i \rfloor$  if  $i$  is even and  $p_i = \lfloor (1 + \gamma)q_i \rfloor$  if  $i$  is odd. Two  $\gamma$  values as 0.2 and 0.8 are considered to generate two different types of instances called  $Z$  and  $W$ , respectively. Note that, all benchmark instances and the demand separation approaches described here are adapted to Karaoglan et al. [Karaoglan et al.2011]. Indeed, this is due to the comparison of the proposed method for CLRP-SPD with the previous work seems reasonable.

The GCM is coded in *MATLAB*<sup>®</sup> 7.0.4 on a computer, holding Intel Xeon 2 GHz equipped with 1 GB RAM. The comparative results are summarized in Tables 2, 3, 4 and 5. The tables show the computational results on Barreto's and Prodhon's sets by applying Salhi and Nagy's and Angelelli and Mansini's demand separation approaches, respectively. Barreto's instances used in Table 1 and 2 include 15 test problems which the number of customers varies between 8 and 100 and the number of candidate depots is between 2 and 15. The name of each Barreto's instance includes the information about the name of the author, who has generated the instance, the publication year of the instance, the number of customers,  $|N_C|$ , and the number of potential depots  $|N_0|$ , (i.e. Author-Year- $|N_C| \times |N_0|$ ). Prodhon's set applied in Table 3 and 4 consists of 22 instances which the number of customers changes between 20, 50 or 100, the number of candidate depots is either 5 or 10, the number of clusters  $clu$  is in  $\{1,2,3\}$  (1 means that all nodes scatter on Euclidean plane), and vehicle capacity  $Q$  is in

{a,b} where  $a = 70$  and  $b = 150$ . Note that, the Prodhon's instances are denoted as  $|N_C| - |N_0| - cluQ$  [Karaoglan et al.2011].

The first column of the Tables 2 to 5 gives the names of CLRP-SPD instances which explained above. The column named *DSS* denotes demand separation strategy. Next two columns summarize computational results for the Branch-and-Cut (B&C) algorithm proposed by Karaoglan et al. [Karaoglan et al.2011]. It is important to note that the B&C algorithm have been performed on Intel Xeon 3.16 GHz equipped with 1 GB RAM computer and a time limit of 4 hours has been imposed on each instance. The next two columns show the solution results of instances for the proposed method explained in this paper. The column labeled *Gap* reports the gap percentage. Note that, the gap percentage for each instance is computed as  $100 \times [(OFV_{GCM} - OFV_{B\&C}) / OFV_{B\&C}]$  where  $OFV_{GCM}$  and  $OFV_{B\&C}$  are denoted as objective function value of GCM and B&C algorithm, respectively. Since to the best our knowledge there is only one approach in this research area, the performance of our approach has been compared to B&C algorithm.

For convenience, Table 6 has summarized the results of Tables 2 to 5. Comparison between the solutions of GCM and B&C algorithm, Table 6 readily reveals that the GCM has improved 95 instances out of 148. Also, while the GCM has solved 14 instances without any changes in solution values, 39 of them have failed to compete with the results of B&C algorithm. Eventually, the proposed GCM is competitive with B&C algorithms in terms of solution quality by providing the 95 new best-known solutions and total average gap of -4.77%. Further performance of the proposed GCM against the B&C algorithm is related to the solving time of instances. In Table 6, it can

be seen that the GCM has solved all instances with the average solving time of 160.86 seconds. But, this value for the B&C algorithm with the mentioned quality of answers is equal to 9221.01 seconds. This comparison easily shows that the GCM has lower running time than the B&C algorithm. So, the GCM can also be considered more efficient than the B&C algorithm in terms of solving time of instances.

Further results of the numerical experiment for evaluating the efficiency of GCM is shown in Table 7. The efficiency of the proposed method is carried out by using 19 standard benchmark test problems of CLRP presented by [Barreto, 2003]. It is noted that, each test problem of CLRP-SPD can be reduced to a CLRP. Actually, if the pickup demands equal 0, then the CLRP-SPD is changed to CLRP. The comparative results are summarized in Table 7. First column of the Table 7 represents the ID number of each test problem. Second column reports the best-known solutions (BKS) that are given in the literature [Lopes et al.2016]. The solutions and CPU times obtained by two approaches: the clustering based heuristic (CH) [Barreto et al. 2007] and HybPSO-LRP [Marinakis and Marinaki, 2008] are shown in next columns. Last column of the table shows the solution and CPU time of the heuristic algorithm.

Results of Table 7 indicate that the heuristic algorithm in comparison of two approaches has been able to obtain 9 best-known solutions out of the 19 test problems. Also, it can be seen that the proposed heuristic method is competitive with two other algorithms in terms of solution quality by providing the lowest average gap (see last row of the Table 7). Consequently, computational results express that the heuristic algorithm is competitive with other algorithms in terms of solution quality.

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**Table 2. Computational results for the instances derived from Barreto's test set by Salhi and Nagy's separation approach.**

| Instances              | DSS | Brach & Cut |                    | GCM     |                    | Gap (%)      |
|------------------------|-----|-------------|--------------------|---------|--------------------|--------------|
|                        |     | OFV         | CPU time (seconds) | OFV     | CPU time (seconds) |              |
| Srivastava86-8×2       | X   | 625.43      | 0.38               | 625.43  | 0.12               | 0.00         |
|                        | Y   | 625.43      | 0.26               | 625.43  | 0.11               | 0.00         |
| Perl83-12×2            | X   | 242.41      | 0.60               | 205.27  | 0.15               | -15.32       |
|                        | Y   | 242.41      | 0.55               | 205.27  | 0.23               | -15.32       |
| Gaskell67-21×5         | X   | 454.48      | 13.34              | 436.22  | 3.54               | -4.02        |
|                        | Y   | 454.48      | 8.98               | 436.22  | 4.21               | -4.02        |
| Gaskell67-22×5         | X   | 629.51      | 109.62             | 608.90  | 4.78               | -3.27        |
|                        | Y   | 629.51      | 64.99              | 608.90  | 5.01               | -3.27        |
| Min92-27×5             | X   | 2998.8      | 1074               | 3051    | 35.65              | 1.74         |
|                        | Y   | 2998.8      | 1440               | 3051.4  | 39.27              | 1.75         |
| Gaskell67-29×5         | X   | 490.34      | 6.63               | 480.67  | 55.45              | -1.97        |
|                        | Y   | 490.34      | 6.57               | 480.67  | 54.34              | -1.97        |
| Gaskell67-32×5_1       | X   | 563.47      | 18.77              | 504.32  | 53.21              | -10.50       |
|                        | Y   | 563.47      | 27.31              | 504.32  | 50.30              | -10.50       |
| Gaskell67-32×5_2       | X   | 507.03      | 13.72              | 504.32  | 61.56              | -0.53        |
|                        | Y   | 507.03      | 7.87               | 504.32  | 55.20              | -0.53        |
| Gaskell67-36×5         | X   | 494.86      | 18.36              | 437.11  | 53.56              | -11.67       |
|                        | Y   | 494.86      | 18.13              | 437.11  | 58.21              | -11.67       |
| Christofides 69-50×5   | X   | 590.53      | 14400              | 568.99  | 77.45              | -3.65        |
|                        | Y   | 592.87      | 14400              | 566.70  | 89.76              | -4.41        |
| Perl83-55×15           | X   | 1014.54     | 14400              | 937.35  | 91.67              | -7.61        |
|                        | Y   | 1017.11     | 14400              | 947.49  | 99.90              | -6.85        |
| Christofides 69-75×10  | X   | 979.91      | 14400              | 837.61  | 189.56             | -14.52       |
|                        | Y   | 962.68      | 14400              | 847.09  | 201.34             | -12.01       |
| Perl83-85×7            | X   | 1462.81     | 14400              | 1354.54 | 237.23             | -7.40        |
|                        | Y   | 1436.52     | 14400              | 1313.50 | 287.41             | -8.56        |
| Daskin95-88×8          | X   | 443.88      | 14400              | 443.88  | 740.67             | 0.00         |
|                        | Y   | 401.98      | 14068.4            | 401.98  | 890.67             | 0.00         |
| Christofides 69-100×10 | X   | 945.75      | 14400              | 857.73  | 955.89             | -9.31        |
|                        | Y   | 935.24      | 14400              | 906.90  | 1024.45            | -3.03        |
| <b>Average</b>         |     |             | <b>5843.28</b>     |         | <b>180.7</b>       | <b>-5.61</b> |

**Table 3. Computational results for the instances derived from Barreto’s test set by Angelelli and Mansini’s separation approach.**

| Instances              | DSS | Brach & Cut |                    | GCM      |                    | Gap (%)      |
|------------------------|-----|-------------|--------------------|----------|--------------------|--------------|
|                        |     | OFV         | CPU time (seconds) | OFV      | CPU time (seconds) |              |
| Srivastava86-8×2       | W   | 873.58      | 0.00               | 873.58   | 0.04               | 0.00         |
|                        | Z   | 806.06      | 0.00               | 806.06   | 0.02               | 0.00         |
| Perl83-12×2            | W   | 243.98      | 0.57               | 205.27   | 0.85               | -15.87       |
|                        | Z   | 243.98      | 0.65               | 205.27   | 0.71               | -15.87       |
| Gaskell67-21×5         | W   | 528.42      | 290.18             | 501.0654 | 6.75               | -5.18        |
|                        | Z   | 513.3       | 89.43              | 512.36   | 7.65               | -0.18        |
| Gaskell67-22×5         | W   | 653.80      | 3.76               | 641.53   | 35.02              | -1.88        |
|                        | Z   | 653.80      | 2.66               | 646.66   | 44.66              | -1.09        |
| Min92-27×5             | W   | 3142.02     | 19.66              | 3182.09  | 44.44              | 1.28         |
|                        | Z   | 3142.02     | 18.12              | 3201.12  | 55.67              | 1.88         |
| Gaskell67-29×5         | W   | 592.1       | 6.76               | 592.1    | 67.89              | 0.00         |
|                        | Z   | 592.1       | 100.91             | 592.1    | 78.09              | 0.00         |
| Gaskell67-32×5_1       | W   | 696.38      | 5694.09            | 659.78   | 46.42              | -5.26        |
|                        | Z   | 643.37      | 259.67             | 618.65   | 53.78              | -3.84        |
| Gaskell67-32×5_2       | W   | 595.27      | 15.68              | 536.92   | 70.09              | -9.80        |
|                        | Z   | 564.33      | 30.90              | 535.68   | 67.76              | -5.08        |
| Gaskell67-36×5         | W   | 540.37      | 27.20              | 475.21   | 60.56              | -12.06       |
|                        | Z   | 540.37      | 46.46              | 469.18   | 73.21              | -13.17       |
| Christofides 69-50×5   | W   | 740.38832   | 14400              | 708.37   | 118.33             | -4.32        |
|                        | Z   | 727.6701    | 14400              | 683.58   | 110.75             | -6.06        |
| Perl83-55×15           | W   | 1609.40     | 14400              | 1278.08  | 56.31              | -20.59       |
|                        | Z   | 1616.80     | 14400              | 1239.93  | 66.3               | -23.31       |
| Christofides 69-75×10  | W   | 1384.3276   | 14400              | 1081.80  | 98.38              | -21.85       |
|                        | Z   | 1250.8598   | 14400              | 1016.65  | 102.34             | -18.72       |
| Perl83-85×7            | W   | 2348.99     | 14400              | 1871.42  | 122.34             | -20.33       |
|                        | Z   | 2336.50     | 14400              | 1871.42  | 111.34             | -19.90       |
| Daskin95-88×8          | W   | 650.50      | 14400              | 593.37   | 775.45             | -8.78        |
|                        | Z   | 563.76      | 14400              | 587.81   | 604.34             | 4.27         |
| Christofides 69-100×10 | W   | 1218.44     | 14400              | 1162.82  | 1425.36            | -4.56        |
|                        | Z   | 1134.40     | 14400              | 1083.86  | 1389.89            | -4.46        |
| <b>Average</b>         |     |             | <b>5980.23</b>     |          | <b>189.82</b>      | <b>-7.82</b> |

**Table 4. Results for instances derived from Prodhon’s test set by Salhi and Nagy’s separation approach.**

| Instances | DSS | Brach & Cut |                    | GCM      |                    | Gap (%) |
|-----------|-----|-------------|--------------------|----------|--------------------|---------|
|           |     | OFV         | CPU time (seconds) | OFV      | CPU time (seconds) |         |
| 20-5-1a   | X   | 16816.50    | 84.51              | 16823.24 | 2.81               | 0.04    |

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|                |   |             |                 |           |               |              |
|----------------|---|-------------|-----------------|-----------|---------------|--------------|
|                | Y | 16816.00    | 73.77           | 16823.24  | 3.41          | 0.04         |
| 20-5-1b        | X | 9167.14     | 1.86            | 9167.14   | 5.63          | 0.00         |
|                | Y | 9167.14     | 0.27            | 9167.14   | 6.21          | 0.00         |
| 20-5-2a        | X | 17814.7     | 27.93           | 17808.40  | 2.14          | -0.04        |
|                | Y | 17814.7     | 16.02           | 17808.40  | 3.56          | -0.04        |
| 20-5-2b        | X | 10257.30    | 1.61            | 10257.30  | 5.32          | 0.00         |
|                | Y | 10257.30    | 1.38            | 10257.30  | 4.9           | 0.00         |
| 50-5-1a        | X | 16377.80    | 14400           | 16403.18  | 62.92         | 0.15         |
|                | Y | 16391.18    | 14400           | 16409.21  | 79.55         | 0.11         |
| 50-5-1b        | X | 13138.15    | 14400           | 13184.45  | 127.78        | 0.35         |
|                | Y | 13138.15    | 14400           | 13168.35  | 134.54        | 0.23         |
| 50-5-2a        | X | 26419.36    | 14400           | 26462.04  | 59.01         | 0.16         |
|                | Y | 26419.09    | 14400           | 26461.13  | 67.6          | 0.16         |
| 50-5-2b        | X | 22268.50    | 213.75          | 22304.80  | 99.5          | 0.16         |
|                | Y | 22268.50    | 1147.76         | 22312.05  | 107.43        | 0.20         |
| 50-5-3a        | X | 11652.09808 | 14400           | 11650.85  | 66.43         | -0.01        |
|                | Y | 11655.6665  | 14400           | 11666.86  | 71.90         | 0.10         |
| 50-5-3b        | X | 8482.56     | 14400           | 8542.07   | 122.45        | 0.70         |
|                | Y | 8472.41     | 14400           | 8512.53   | 132.56        | 0.47         |
| 100-5-1a       | X | 102572.30   | 14400           | 101660.86 | 88.45         | -0.89        |
|                | Y | 102555.05   | 14400           | 101659.05 | 104.65        | -0.87        |
| 100-5-1b       | X | 94997.86    | 14400           | 94041.51  | 211.65        | -1.01        |
|                | Y | 94992.65    | 14400           | 94045.16  | 241.12        | -1.00        |
| 100-5-2a       | X | 105771.22   | 14400           | 104781.10 | 98.11         | -0.94        |
|                | Y | 105771.22   | 14400           | 104806.90 | 90.76         | -0.91        |
| 100-5-2b       | X | 97291.57104 | 14400           | 96304.12  | 289.06        | -1.01        |
|                | Y | 97274.0442  | 14400           | 96390.14  | 255.6         | -0.91        |
| 100-5-3a       | X | 56648.23857 | 14400           | 57699.55  | 79.98         | 1.86         |
|                | Y | 56706.38018 | 14400           | 57681.65  | 95.39         | 1.72         |
| 100-5-3b       | X | 50279.84706 | 14400           | 50305.90  | 266.89        | 0.05         |
|                | Y | 50265.99872 | 14400           | 50305.70  | 255.11        | 0.08         |
| 100-10-1a      | X | 110959.70   | 14400           | 110063.57 | 110.56        | -0.81        |
|                | Y | 110961.72   | 14400           | 110086.83 | 123.3         | -0.79        |
| 100-10-1b      | X | 102511.94   | 14400           | 102580.00 | 301.23        | 0.07         |
|                | Y | 102497.70   | 14400           | 102581.56 | 298.5         | 0.08         |
| 100-10-2a      | X | 204974.95   | 14400           | 105740.61 | 130.34        | -48.41       |
|                | Y | 109639.16   | 14400           | 105708.27 | 125.45        | -3.59        |
| 100-10-2b      | X | 100210.81   | 14400           | 98255.80  | 310.89        | -1.95        |
|                | Y | 100209.09   | 14400           | 98304.60  | 331.23        | -1.90        |
| 100-10-3a      | X | 101819.79   | 14400           | 99963.24  | 145.69        | -1.82        |
|                | Y | 100243.3149 | 14400           | 99849.24  | 130.13        | -0.39        |
| 100-10-3b      | X | 93515.92    | 14400           | 93522.97  | 335.1         | 0.01         |
|                | Y | 93568.48    | 14400           | 93537.50  | 368.8         | -0.03        |
| <b>Average</b> |   |             | <b>11162.93</b> |           | <b>135.31</b> | <b>-1.38</b> |

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**Table 5. Results for instances derived from Prodhon's test set by Angelelli and Mansini's separation.**

| Instances      | DSS | Brach & Cut |                    | GCM       |                    | Gap (%)     |
|----------------|-----|-------------|--------------------|-----------|--------------------|-------------|
|                |     | OFV         | CPU time (seconds) | OFV       | CPU time (seconds) |             |
| 20-5-1a        | W   | 26457.70    | 232.48             | 26512.86  | 10.21              | 0.21        |
|                | Z   | 26461.30    | 75.24              | 26494.23  | 8.14               | 0.12        |
| 20-5-1b        | W   | 18718.80    | 3.22               | 18721.68  | 11.67              | 0.02        |
|                | Z   | 18703.00    | 0.94               | 18715.12  | 12.56              | 0.06        |
| 20-5-2a        | W   | 27988.7     | 37.79              | 27988.30  | 7.13               | 0.00        |
|                | Z   | 27980.6     | 73.81              | 27988.30  | 6.98               | 0.03        |
| 20-5-2b        | W   | 17125.50    | 7.09               | 17125.46  | 6.43               | 0.00        |
|                | Z   | 17120.50    | 4.4                | 17125.46  | 7.8                | 0.03        |
| 50-5-1a        | W   | 37935.14536 | 14400              | 32870.09  | 70.21              | -13.35      |
|                | Z   | 32902.9135  | 14400              | 32803.58  | 68.33              | -0.30       |
| 50-5-1b        | W   | 26597.13694 | 14400              | 26551.14  | 155.67             | -0.17       |
|                | Z   | 26578.55544 | 14400              | 26546.76  | 133.8              | -0.12       |
| 50-5-2a        | W   | 43949.05416 | 14400              | 42980.40  | 50.4               | -2.20       |
|                | Z   | 41892.72128 | 14400              | 41915.76  | 58.1               | 0.05        |
| 50-5-2b        | W   | 35708.26047 | 14400              | 35717.43  | 122.65             | 0.03        |
|                | Z   | 35687.61324 | 14400              | 35698.53  | 147.11             | 0.03        |
| 50-5-3a        | W   | 24627.39    | 14400              | 23599.09  | 87.9               | -4.18       |
|                | Z   | 23562.60    | 14400              | 23538.76  | 65.59              | -0.10       |
| 50-5-3b        | W   | 17218.0822  | 14400              | 17212.90  | 150.45             | -0.03       |
|                | Z   | 17196.42633 | 14400              | 17202.49  | 112.23             | 0.04        |
| 100-5-1a       | W   | 162456.77   | 14400              | 160551.90 | 159.7              | -1.17       |
|                | Z   | 160362.16   | 14400              | 159354.18 | 134.69             | -0.63       |
| 100-5-1b       | W   | 146833.44   | 14400              | 145828.19 | 289.14             | -0.68       |
|                | Z   | 146756.94   | 14400              | 144782.58 | 301.54             | -1.35       |
| 100-5-2a       | W   | 215851.62   | 14400              | 168570.32 | 86.28              | -21.90      |
|                | Z   | 224798.59   | 14400              | 166588.13 | 102.73             | -25.89      |
| 100-5-2b       | W   | 199313.6439 | 14400              | 152916.61 | 277.29             | -23.28      |
|                | Z   | 199216.808  | 14400              | 152840.26 | 293.51             | -23.28      |
| 100-5-3a       | W   | 118696.87   | 14400              | 113745.43 | 119.37             | -4.17       |
|                | Z   | 114592.63   | 14400              | 112575.12 | 102.31             | -1.76       |
| 100-5-3b       | W   | 102136.13   | 14400              | 99170.07  | 288.98             | -2.90       |
|                | Z   | 100071.09   | 14400              | 98131.84  | 312.69             | -1.94       |
| 100-10-1a      | W   | 227609.67   | 14400              | 224736.17 | 99.74              | -1.26       |
|                | Z   | 223612.90   | 14400              | 222638.50 | 115.14             | -0.44       |
| 100-10-1b      | W   | 209293.08   | 14400              | 209111.21 | 345.67             | -0.09       |
|                | Z   | 209041.58   | 14400              | 209094.28 | 330.39             | 0.03        |
| 100-10-2a      | W   | 267097.085  | 14400              | 220384.46 | 134.78             | -17.49      |
|                | Z   | 267173.0806 | 14400              | 220867.67 | 145.71             | -17.33      |
| 100-10-2b      | W   | 162394.08   | 14400              | 169347.72 | 378.89             | 4.28        |
|                | Z   | 248472.16   | 14400              | 169333.79 | 380.21             | -31.85      |
| 100-10-3a      | W   | 254277.12   | 14400              | 178859.35 | 148.34             | -29.66      |
|                | Z   | 170464.48   | 14400              | 177956.21 | 131.23             | 4.39        |
| 100-10-3b      | W   | 238317.47   | 14400              | 165533.13 | 388.65             | -30.54      |
|                | Z   | 154172.07   | 14400              | 164617.22 | 377.42             | 6.77        |
| <b>Average</b> |     |             | <b>11791.7</b>     |           | <b>153.13</b>      | <b>-5.5</b> |

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**Table 6. Summarized of the computational results between GCM and B&C algorithm.**

| Test set       | DSS | No. of instances | Statues of GCM results against B&C |            |           | Average of CPU time (seconds) |               | Average of Gap (%) |
|----------------|-----|------------------|------------------------------------|------------|-----------|-------------------------------|---------------|--------------------|
|                |     |                  | Improved                           | Unchanging | Worsened  | Brach & Cut                   | GCM           |                    |
| Barreto        | X   | 15               | 24                                 | 4          | 2         | 5843.28                       | 180.7         | -5.61              |
|                | Y   | 15               |                                    |            |           |                               |               |                    |
| Barreto        | W   | 15               | 23                                 | 4          | 3         | 5980.23                       | 189.82        | -7.82              |
|                | Z   | 15               |                                    |            |           |                               |               |                    |
| Prodhon        | X   | 22               | 20                                 | 4          | 20        | 11162.93                      | 135.31        | -1.38              |
|                | Y   | 22               |                                    |            |           |                               |               |                    |
| Prodhon        | W   | 22               | 28                                 | 2          | 14        | 11791.7                       | 153.13        | -5.5               |
|                | Z   | 22               |                                    |            |           |                               |               |                    |
| <b>Total</b>   |     | <b>148</b>       | <b>95</b>                          | <b>14</b>  | <b>39</b> |                               |               |                    |
| <b>Average</b> |     |                  |                                    |            |           | <b>9221.01</b>                | <b>160.86</b> | <b>-4.77</b>       |

**Table 7. Computational results of heuristic algorithm on standard test problems of CLRP.**

| CLRP instance   | BKS      | CH           |             | HybPSO-LRP   |             | GCM             |             |
|-----------------|----------|--------------|-------------|--------------|-------------|-----------------|-------------|
|                 |          | Solution     | Gap (%)     | Solution     | Gap (%)     | Solution        | Gap (%)     |
| Christ69–50×5   | 565.6    | 582.7        | 3.02        | 582.7        | 3.02        | <b>565.6</b>    | 0.00        |
| Christ69–75×10  | 844.4    | 886.3        | 4.96        | 886.3        | 4.96        | 886.3           | 4.96        |
| Christ69–100×10 | 833.9    | 889.4        | 6.66        | 889.4        | 6.66        | <b>833.9</b>    | 0.00        |
| Gaskell67–21×5  | 424.9    | 435.9        | 2.59        | 432.9        | 1.88        | 427.7           | 0.66        |
| Gaskell67–22×5  | 585.1    | 591.5        | 1.09        | 588.5        | 0.58        | 591.5           | 1.09        |
| Gaskell67–29×5  | 512.1    | <b>512.1</b> | 0.00        | <b>512.1</b> | 0.00        | <b>512.1</b>    | 0.00        |
| Gaskell67–32×5  | 562.2    | 571.7        | 1.69        | 570.8        | 1.53        | <b>562.2</b>    | 0.00        |
| Gaskell67–32×5  | 504.3    | 511.4        | 1.41        | 511.1        | 1.35        | <b>504.3</b>    | 0.00        |
| Gaskell67–36×5  | 460.4    | 470.7        | 2.24        | 470.7        | 2.24        | 469.2           | 1.91        |
| Perl83–12×2     | 204      | <b>204</b>   | 0.00        | <b>204</b>   | 0.00        | 205.3           | 0.64        |
| Perl83–55×15    | 1112.1   | 1136.2       | 2.17        | 1135.9       | 2.14        | 1127.1          | 1.35        |
| Perl83–85×7     | 1622.5   | 1656.9       | 2.12        | 1656.9       | 2.12        | <b>1622.5</b>   | 0.00        |
| Perl83–318×4    | 557275.2 | 580680.2     | 4.20        | 580680.2     | 4.20        | 580791.5        | 4.22        |
| Perl83–318×4    | 670118.5 | 747619       | 11.57       | 747619       | 11.57       | <b>670118.5</b> | 0.00        |
| Dasnki95–88×8   | 355.8    | 384.9        | 8.18        | 384.9        | 8.18        | 384.9           | 8.18        |
| Dasnki95–150×10 | 43919.9  | 46642.7      | 6.20        | 46642.7      | 6.20        | 46642.7         | 6.20        |
| Min92–27×5      | 3062     | <b>3062</b>  | 0.00        | <b>3062</b>  | 0.00        | <b>3062</b>     | 0.00        |
| Min92–134×8     | 5709     | 6238         | 9.27        | 6230         | 9.13        | 6229            | 9.11        |
| Or76–117×14     | 12290.3  | 12474.2      | 1.50        | 12474.2      | 1.50        | <b>12290.3</b>  | 0.00        |
| <b>Avg.</b>     |          |              | <b>3.62</b> |              | <b>3.54</b> |                 | <b>2.02</b> |

Gap: relative percentage gap calculated as  $\%100 \times (\text{solution values obtained by algorithm} - \text{BKS})/\text{BKS}$ .  
 Bold numbers indicate that best known solution values are attained by the corresponding approach.



## 5. Conclusion

Logistics costs often represent a large part of the cost of companies. In order to reduce them, facility location and vehicle routing are crucial. In the management decision of the logistics, facility location problems and vehicle routing problems are interdependent. But often, they are considered separately and sometimes increase the total cost. This paper contributes to the capacitated location-routing problem with simultaneous pickup and delivery. A node-based MIP formulation for the CLRP-SPD based on Karaoglan et al. [Karaoglan et al.2012] is proposed. To solve the problem, a GCM with four phases was proposed where greedy search algorithm was applied to cluster the customers in first phase. Next phase determined the gravity centers of cluster to select the appropriate depot(s). Clusters of customer were assigned to selected depot(s) in the third phase. In the fourth phase the routes between depot(s) and assigned clusters were built by ant colony system. Comparisons of the results of the GCM with the B&C algorithm obtained from the literature of the CLRP-SPD showed that the efficiency of the proposed method was satisfactory. In 95 instances out of 148, the GCM found better solutions. While the total average gap of instances was -4.77%, the GCM also solved all instances in lower solving time compared to the B&C algorithm. Finally, it is concluded that the GCM is more effective than the B&C algorithm in terms of solution quality and solving time of instances. This paper has some capable future research directions: considering the CLRP-SPD with fuzzy pickup and delivery demands, developing other solution algorithms e.g. hybrid evolutionary algorithms, and developing the model by some more realistic assumptions e.g. heterogeneous vehicles with unequal capacities.

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